MATH 413 - ASSIGNMENT 1

Due **January 25th**, on paper at the beginning of class.
See the syllabus for expectations regarding homework assignments.
The assignment is worth 20 points. Assignments typed in in LaTeXwill receive a 2 point bonus.

Give a full written solution to each of the following questions:

(1) Let $n \geq 4$ be a positive even integer. n seats are arranged in a circle, labelled $1, \ldots n$ in circular order.
- In how many ways can Alice and Bob both choose a seat if Bob refuses to sit next to Alice?
- In how many ways can Alice and Bob both choose a seat if Bob refuses to sit next to Alice and also refuses to sit on an even-numbered seat?

(2) Use the Division Principle to prove that the letters in COMBINATORICS can be rearranged into $13!/8$ distinct words of length 13.

(3) Suppose you have 13 tiles, each with one letter, and that the letters on these tiles spell COMBINATORICS. How many distinct two-letter words can be made from these tiles? (Note that the same tile cannot be used twice, but two tiles with the same letter can be used together.)

(4) Let n and k be positive integers, with $n \geq 3$ and $n \geq k$. Recall that $\binom{n}{k}$ denotes the number of subsets of $\{1, \ldots, n\}$ of size k.
- Use the Addition Principle to prove that
$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}.$$
- Use the Addition Principle to prove that
$$\binom{n}{3} = \sum_{i=1}^{n-2} \binom{n-i}{2}.$$

Note: Your solution should not use the identity $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.