MATH 413 - ASSIGNMENT 2

Due **February 1st**, on Gradescope at the beginning of class.
See the syllabus for expectations regarding homework assignments.
The assignment is worth 20 points. Assignments typed in in LaTeXwill receive a 2 point bonus.

Give a full written solution to each of the following questions:

(1) In how many ways can six identical rooks be placed on an 8×8 chessboard so that no two rooks can attack each other? (Note: Two rooks can attack each other if and only if they occupy the same row or column of the chessboard.)

(2) Let n and k be positive integers, with $n \geq k \geq 1$. Compute the number of k-permutations of $\{1, \ldots, n\}$ containing the element n.

(3) Let $n \geq m$ be positive integers. Prove that

$$\sum_{k=0}^{n} \binom{k}{m} \binom{n}{k} = \binom{n}{m} 2^{n-m}.$$

(Hint: This can be solved with a committee selection argument.)

(4) Prove that the number of binary sequences containing m 0's and at most n 1's is

$$\binom{m+n+1}{m+1}.$$

(5) Let n be a positive integer. Prove that

$$\binom{3n+1}{n} = \sum_{k=0}^{n} \binom{n+k}{k} \binom{2n-k}{n-k}.$$

(Hint: Use lattice paths. Where does a path complete half of its horizontal distance?)