Math 413

Brualdi Chapter 2

Binomial coefficients

- The *binomial coefficient* $\binom{n}{k}$ is the number of subsets of $\{1, \ldots, n\}$ of size k.
- $\binom{n}{k}$ is can be written C_k^n or $C(n, k)$ or ${}_nC_k$.
- $\binom{n}{k} = \frac{n!}{k!(n-k)!}$.

Theorem (Multiplication principle)

Let S be a finite set of ordered k-tuples $(a_1, \ldots, a_k)$, where

- *a_1 is chosen from a set of p_1 objects*
- *a_2 is chosen from a set of p_2 objects*
- $\ldots$
- *a_k is chosen from a set of p_k objects.*

Then $|S| = p_1 \ldots p_k$.

Example

How many ways are there to form a 3-letter word using $\{a, b, c, d, e, f\}$,

with repetition allowed?

Example

How many ways are there to form a 3-letter word using $\{a, b, c, d, e, f\}$,

with no repetition?

Example

How many ways are there to form a 3-letter word using $\{a, b, c, d, e, f\}$,

with no repetition and containing the letter 'e'?

Example

How many ways are there to form a 3-letter word using $\{a, b, c, d, e, f\}$,

with repetition and containing the letter 'e'?

Slightly harder example

How many 8-digit passwords can be made using the letters A - Z such that

- 3 letters are uppercase,
- 5 letters are lowercase?

Theorem (Division principle)

Let S be a finite set. If $S_1, \ldots, S_k$ is a partition of S, and if $|S_i| = t$ for each $i \in \{1, \ldots, k\}$, then $k = |S|/t$.

Example

Show that the letters in PUPPY can be rearranged into 20 distinct words of length 5,

using the Division Principle.

Example

Show that the letters in PUPPY can be rearranged into 20 distinct words of length 5,

using the Multiplication Principle.