MATH 484 - ASSIGNMENT 1

Due **January 24**, on Gradescope at 11:59pm Central Time.
See the syllabus for expectations regarding homework assignments.
The assignment is worth 40 points. Assignments typed in in LaTeXwill receive a 4 point bonus.

Give a full written solution to each of the following questions:

(1) (10 points) Find the global and local minimizers of the following functions. For each minimizer, state whether it is a strict global/local minimizer. (You do not need to write a proof, but please provide some plain-language explanation.)
- $f(x) = x^4 - 32x$
- $f(x) = \lfloor x^2 \rfloor$
- $f(x) = (x-2)^2 + |x|$.

(2) (10 points) Let V be a vector space equipped with the field $\mathbb{R}$ and an inner product $\cdot : V \times V \to \mathbb{R}$, and let $\mathbf{u}, \mathbf{v} \in V$ be two vectors. Prove that $\mathbf{u} \cdot \mathbf{v} = 0$ if and only if $\|\mathbf{u}\| \leq \|\mathbf{u} + c\,\mathbf{v}\|$ for every real number $c \in \mathbb{R}$.

(3) (10 points) For a function $f : \mathbb{R} \to \mathbb{R}$ and a real number $c \in \mathbb{R}$, define the *sublevel set* f_c^- as

$$f_c^- = \{x \in \mathbb{R} : f(x) \leq c\}.$$

Prove that f is coercive if and only if for every $c \in \mathbb{R}$, there exists a radius $R > 0$ such that $f_c^- \subseteq (-R, R)$, where $(-R, R)$ is the open real interval from $-R$ to R.

(4) (10 points) Let $x_1, x_2, \ldots$ be a convergent sequence of real numbers. Recall that for $L \in \mathbb{R}$, we say that

$$\lim_{k\to\infty} x_k = L$$

if and only if for every $\varepsilon > 0$, there exists a natural number N such that whenever $k \geq N$, $|x_k - L| < \varepsilon$.
- Using this definition of a limit, prove that if $x_k > 0$ whenever k is even and $x_k < 0$ whenever k is odd, then $\lim_{k\to\infty} x_k = 0$.
- Give an example of a convergent sequence that satisfies the properties described in this question.